

Supplementary Document of SNPRuler

Xiang Wan¹, Can Yang¹, Qiang Yang², Hong Xue³, Nelson L.S. Tang⁴ and Weichuan Yu¹

¹Department of Electronic and Computer Engineering, The Hong Kong University of Science and Technology.

² Department of Computer Science, The Hong Kong University of Science and Technology.

³Department of Biochemistry, The Hong Kong University of Science and Technology.

⁴Laboratory for Genetics of Disease Susceptibility, Li Ka Shing
Institute of Health Sciences, The Chinese University of Hong Kong.

1 Models used in simulation studies

1.1 Models with marginal effects

Table 1 lists the three 2-locus epistatic models used in Case 1 of Section 3.1. Table 2 shows the 3-locus epistatic model, which is used as Model 4 in Case 1 of Section 3.1.

| Model 1 | AA | Aa | aa |
|---------|------------------|------------------|------------------|
| BB | 1 | $1 + \theta$ | $(1 + \theta)^2$ |
| Bb | $1 + \theta$ | $(1 + \theta)^2$ | $(1 + \theta)^3$ |
| bb | $(1 + \theta)^2$ | $(1 + \theta)^3$ | $(1 + \theta)^4$ |
| Model 2 | AA | Aa | aa |
| BB | 1 | 1 | 1 |
| Bb | 1 | $(1 + \theta)^2$ | $(1 + \theta)^3$ |
| bb | 1 | $(1 + \theta)^3$ | $(1 + \theta)^4$ |
| Model 3 | AA | Aa | aa |
| BB | 1 | 1 | 1 |
| Bb | 1 | $(1 + \theta)$ | $(1 + \theta)$ |
| bb | 1 | $(1 + \theta)$ | $(1 + \theta)$ |

Table 1: Three models with marginal effects, which are proposed in [1] and also used in [4] and our paper.

| | AABB | AABb | AAbb | AaBB | AaBb | Aabb | aaBB | aaBb | aabb |
|----|------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|------|
| BB | 1 | 1 | 1 | 1 | 1 | $1 + \theta$ | 1 | $1 + \theta$ | 1 |
| Bb | 1 | 1 | $1 + \theta$ | 1 | $1 + \theta$ | 1 | $1 + \theta$ | 1 | 1 |
| bb | 1 | $1 + \theta$ | 1 | $1 + \theta$ | 1 | 1 | 1 | 1 | 1 |

Table 2: The risk odds in a 3-locus epistatic model used in [4] and our paper.

1.2 Parameter setting for data generation in Case 1 of simulation experiments

The data generation follows the same procedure in [4]. The marginal effects of disease loci are controlled by the parameter λ (readers may check [4] for details). The Linkage Disequilibrium (LD) between SNPs is measured by r^2 . The following settings are chosen for the practical concerns as discussed in [3]:

- Sample size: 2000 samples (1000 cases and 1000 controls) and 4000 samples (2000 cases and 2000 controls) are simulated under each parameter setting.
- *MAF*: *MAF* (minor allele frequency) is chosen from 0.1 to 0.5.
- Marginal effect size λ : $\lambda = 0.3$ for Model 1 and $\lambda = 0.2$ for Model 2, 3 and 4. The value of θ in each model is computed from the specified λ .
- Linkage Disequilibrium (LD) r^2 : $r^2 = 1$ for directly genotyped disease loci and $r^2 = 0.7$ for genotyped LD markers.
- SNP size: 2000 SNP markers are simulated in each setting.

2 Models without marginal effects

The epistatic models without marginal effects used in simulation experiments are given in Table 3-8.

| $h^2 = 0.4, MAF = 0.2$ | | | | $h^2 = 0.4, MAF = 0.4$ | | | |
|------------------------|-------|-------|-------|------------------------|-------|-------|-------|
| Model epi1 | AA | Aa | aa | Model epi6 | AA | Aa | aa |
| BB | 0.486 | 0.960 | 0.538 | BB | 0.077 | 0.656 | 0.880 |
| Bb | 0.947 | 0.004 | 0.811 | Bb | 0.892 | 0.235 | 0.312 |
| bb | 0.640 | 0.606 | 0.909 | bb | 0.174 | 0.842 | 0.106 |
| $h^2 = 0.4, MAF = 0.2$ | | | | $h^2 = 0.4, MAF = 0.4$ | | | |
| Model epi2 | AA | Aa | aa | Model epi7 | AA | Aa | aa |
| BB | 0.469 | 0.956 | 0.697 | BB | 0.895 | 0.323 | 0.161 |
| Bb | 0.945 | 0.019 | 0.585 | Bb | 0.068 | 0.728 | 0.806 |
| bb | 0.786 | 0.407 | 0.013 | bb | 0.925 | 0.233 | 0.362 |
| $h^2 = 0.4, MAF = 0.2$ | | | | $h^2 = 0.4, MAF = 0.4$ | | | |
| Model epi3 | AA | Aa | aa | Model epi8 | AA | Aa | aa |
| BB | 0.498 | 0.954 | 0.786 | BB | 0.805 | 0.251 | 0.085 |
| Bb | 0.978 | 0.038 | 0.428 | Bb | 0.002 | 0.668 | 0.638 |
| bb | 0.590 | 0.821 | 0.380 | bb | 0.830 | 0.079 | 0.542 |
| $h^2 = 0.4, MAF = 0.2$ | | | | $h^2 = 0.4, MAF = 0.4$ | | | |
| Model epi4 | AA | Aa | aa | Model epi9 | AA | Aa | aa |
| BB | 0.505 | 0.988 | 0.624 | BB | 0.307 | 0.682 | 0.958 |
| Bb | 0.945 | 0.085 | 0.807 | Bb | 0.997 | 0.390 | 0.281 |
| bb | 0.969 | 0.116 | 0.159 | bb | 0.012 | 0.990 | 0.698 |
| $h^2 = 0.4, MAF = 0.2$ | | | | $h^2 = 0.4, MAF = 0.4$ | | | |
| Model epi5 | AA | Aa | aa | Model epi10 | AA | Aa | aa |
| BB | 0.486 | 0.963 | 0.512 | BB | 0.083 | 0.891 | 0.037 |
| Bb | 0.941 | 0.006 | 0.899 | Bb | 0.619 | 0.271 | 0.691 |
| bb | 0.691 | 0.541 | 0.614 | bb | 0.853 | 0.079 | 0.742 |

Table 3: Models with $h^2 = 0.4, MAF = 0.2, 0.4$.

| $h^2 = 0.3, MAF = 0.2$ | | | | $h^2 = 0.3, MAF = 0.4$ | | | |
|------------------------|-------|-------|-------|------------------------|-------|-------|-------|
| Model epi11 | AA | Aa | aa | Model epi16 | AA | Aa | aa |
| BB | 0.500 | 0.926 | 0.615 | BB | 0.891 | 0.362 | 0.480 |
| Bb | 0.895 | 0.131 | 0.647 | Bb | 0.213 | 0.829 | 0.601 |
| bb | 0.858 | 0.160 | 0.999 | bb | 0.925 | 0.267 | 0.685 |
| $h^2 = 0.3, MAF = 0.2$ | | | | $h^2 = 0.3, MAF = 0.4$ | | | |
| Model epi12 | AA | Aa | aa | Model epi17 | AA | Aa | aa |
| BB | 0.413 | 0.851 | 0.535 | BB | 0.077 | 0.689 | 0.417 |
| Bb | 0.831 | 0.008 | 0.580 | Bb | 0.763 | 0.150 | 0.491 |
| bb | 0.692 | 0.268 | 0.736 | bb | 0.196 | 0.657 | 0.247 |
| $h^2 = 0.3, MAF = 0.2$ | | | | $h^2 = 0.3, MAF = 0.4$ | | | |
| Model epi13 | AA | Aa | aa | Model epi18 | AA | Aa | aa |
| BB | 0.455 | 0.848 | 0.897 | BB | 0.132 | 0.793 | 0.274 |
| Bb | 0.890 | 0.088 | 0.016 | Bb | 0.799 | 0.213 | 0.514 |
| bb | 0.562 | 0.686 | 0.467 | bb | 0.255 | 0.528 | 0.793 |
| $h^2 = 0.3, MAF = 0.2$ | | | | $h^2 = 0.3, MAF = 0.4$ | | | |
| Model epi14 | AA | Aa | aa | Model epi19 | AA | Aa | aa |
| BB | 0.609 | 0.980 | 0.980 | BB | 0.611 | 0.104 | 0.759 |
| Bb | 0.993 | 0.300 | 0.275 | Bb | 0.180 | 0.674 | 0.019 |
| bb | 0.876 | 0.483 | 0.683 | bb | 0.532 | 0.189 | 0.681 |
| $h^2 = 0.3, MAF = 0.2$ | | | | $h^2 = 0.3, MAF = 0.4$ | | | |
| Model epi15 | AA | Aa | aa | Model epi20 | AA | Aa | aa |
| BB | 0.486 | 0.963 | 0.512 | BB | 0.091 | 0.827 | 0.863 |
| Bb | 0.941 | 0.006 | 0.899 | Bb | 0.869 | 0.393 | 0.415 |
| bb | 0.691 | 0.541 | 0.614 | bb | 0.738 | 0.508 | 0.363 |

Table 4: Models with $h^2 = 0.3, MAF = 0.2, 0.4$.

| $h^2 = 0.2, MAF = 0.2$ | | | | $h^2 = 0.2, MAF = 0.4$ | | | |
|------------------------|-------|-------|-------|------------------------|-------|-------|-------|
| Model epi21 | AA | Aa | aa | Model epi26 | AA | Aa | aa |
| BB | 0.428 | 0.757 | 0.812 | BB | 0.356 | 0.891 | 0.809 |
| Bb | 0.788 | 0.132 | 0.044 | Bb | 0.955 | 0.508 | 0.611 |
| bb | 0.559 | 0.548 | 0.373 | bb | 0.617 | 0.755 | 0.630 |
| $h^2 = 0.2, MAF = 0.2$ | | | | $h^2 = 0.2, MAF = 0.4$ | | | |
| Model epi22 | AA | Aa | aa | Model epi27 | AA | Aa | aa |
| BB | 0.507 | 0.842 | 0.605 | BB | 0.086 | 0.536 | 0.641 |
| Bb | 0.845 | 0.162 | 0.629 | Bb | 0.677 | 0.275 | 0.096 |
| bb | 0.581 | 0.678 | 0.729 | bb | 0.219 | 0.413 | 0.712 |
| $h^2 = 0.2, MAF = 0.2$ | | | | $h^2 = 0.2, MAF = 0.4$ | | | |
| Model epi23 | AA | Aa | aa | Model epi28 | AA | Aa | aa |
| BB | 0.577 | 0.247 | 0.428 | BB | 0.855 | 0.339 | 0.772 |
| Bb | 0.227 | 0.928 | 0.578 | Bb | 0.513 | 0.651 | 0.607 |
| bb | 0.586 | 0.262 | 0.158 | bb | 0.250 | 0.999 | 0.154 |
| $h^2 = 0.2, MAF = 0.2$ | | | | $h^2 = 0.2, MAF = 0.4$ | | | |
| Model epi24 | AA | Aa | aa | Model epi29 | AA | Aa | aa |
| BB | 0.340 | 0.637 | 0.654 | BB | 0.506 | 0.838 | 0.024 |
| Bb | 0.689 | 0.017 | 0.041 | Bb | 0.603 | 0.454 | 0.957 |
| bb | 0.242 | 0.866 | 0.403 | bb | 0.729 | 0.427 | 0.753 |
| $h^2 = 0.2, MAF = 0.2$ | | | | $h^2 = 0.2, MAF = 0.4$ | | | |
| Model epi25 | AA | Aa | aa | Model epi30 | AA | Aa | aa |
| BB | 0.387 | 0.726 | 0.734 | BB | 0.393 | 0.764 | 0.664 |
| Bb | 0.749 | 0.090 | 0.034 | Bb | 0.850 | 0.398 | 0.733 |
| bb | 0.551 | 0.401 | 0.724 | bb | 0.406 | 0.927 | 0.147 |

Table 5: Models with $h^2 = 0.2, MAF = 0.2, 0.4$.

| $h^2 = 0.1, MAF = 0.2$ | | | | $h^2 = 0.1, MAF = 0.4$ | | | |
|------------------------|-------|-------|-------|------------------------|-------|-------|-------|
| Model epi31 | AA | Aa | aa | Model epi36 | AA | Aa | aa |
| BB | 0.463 | 0.703 | 0.431 | BB | 0.137 | 0.484 | 0.187 |
| Bb | 0.653 | 0.277 | 0.806 | Bb | 0.482 | 0.166 | 0.365 |
| bb | 0.830 | 0.008 | 0.129 | bb | 0.193 | 0.361 | 0.430 |
| $h^2 = 0.1, MAF = 0.2$ | | | | $h^2 = 0.1, MAF = 0.4$ | | | |
| Model epi32 | AA | Aa | aa | Model epi37 | AA | Aa | aa |
| BB | 0.319 | 0.507 | 0.569 | BB | 0.469 | 0.198 | 0.754 |
| Bb | 0.553 | 0.105 | 0.045 | Bb | 0.337 | 0.502 | 0.141 |
| bb | 0.203 | 0.777 | 0.280 | bb | 0.339 | 0.453 | 0.285 |
| $h^2 = 0.1, MAF = 0.2$ | | | | $h^2 = 0.1, MAF = 0.4$ | | | |
| Model epi33 | AA | Aa | aa | Model epi38 | AA | Aa | aa |
| BB | 0.627 | 0.393 | 0.335 | BB | 0.478 | 0.311 | 0.864 |
| Bb | 0.396 | 0.779 | 0.953 | Bb | 0.387 | 0.579 | 0.263 |
| bb | 0.174 | 0.842 | 0.106 | bb | 0.634 | 0.436 | 0.138 |
| $h^2 = 0.1, MAF = 0.2$ | | | | $h^2 = 0.1, MAF = 0.4$ | | | |
| Model epi34 | AA | Aa | aa | Model epi39 | AA | Aa | aa |
| BB | 0.297 | 0.540 | 0.441 | BB | 0.068 | 0.299 | 0.017 |
| Bb | 0.541 | 0.072 | 0.278 | Bb | 0.289 | 0.044 | 0.285 |
| bb | 0.434 | 0.293 | 0.228 | bb | 0.048 | 0.262 | 0.174 |
| $h^2 = 0.1, MAF = 0.2$ | | | | $h^2 = 0.1, MAF = 0.4$ | | | |
| Model epi35 | AA | Aa | aa | Model epi40 | AA | Aa | aa |
| BB | 0.332 | 0.562 | 0.573 | BB | 0.539 | 0.120 | 0.258 |
| Bb | 0.583 | 0.112 | 0.147 | Bb | 0.165 | 0.378 | 0.325 |
| bb | 0.399 | 0.496 | 0.033 | bb | 0.123 | 0.426 | 0.276 |

Table 6: Models with $h^2 = 0.1, MAF = 0.2, 0.4$.

| $h^2 = 0.05, MAF = 0.2$ | | | | $h^2 = 0.05, MAF = 0.4$ | | | |
|-------------------------|-------|-------|-------|-------------------------|-------|-------|-------|
| Model epi41 | AA | Aa | aa | Model epi46 | AA | Aa | aa |
| BB | 0.492 | 0.664 | 0.481 | BB | 0.002 | 0.155 | 0.214 |
| Bb | 0.642 | 0.330 | 0.746 | Bb | 0.199 | 0.071 | 0.022 |
| bb | 0.656 | 0.396 | 0.000 | bb | 0.081 | 0.122 | 0.135 |
| $h^2 = 0.05, MAF = 0.2$ | | | | $h^2 = 0.05, MAF = 0.4$ | | | |
| Model epi42 | AA | Aa | aa | Model epi47 | AA | Aa | aa |
| BB | 0.499 | 0.639 | 0.765 | BB | 0.188 | 0.020 | 0.171 |
| Bb | 0.666 | 0.389 | 0.083 | Bb | 0.032 | 0.174 | 0.059 |
| bb | 0.543 | 0.527 | 0.953 | bb | 0.134 | 0.087 | 0.092 |
| $h^2 = 0.05, MAF = 0.2$ | | | | $h^2 = 0.05, MAF = 0.4$ | | | |
| Model epi43 | AA | Aa | aa | Model epi48 | AA | Aa | aa |
| BB | 0.212 | 0.350 | 0.116 | BB | 0.005 | 0.179 | 0.251 |
| Bb | 0.336 | 0.054 | 0.495 | Bb | 0.211 | 0.100 | 0.026 |
| bb | 0.227 | 0.273 | 0.495 | bb | 0.156 | 0.098 | 0.156 |
| $h^2 = 0.05, MAF = 0.2$ | | | | $h^2 = 0.05, MAF = 0.4$ | | | |
| Model epi44 | AA | Aa | aa | Model epi49 | AA | Aa | aa |
| BB | 0.805 | 0.683 | 0.638 | BB | 0.174 | 0.321 | 0.154 |
| Bb | 0.657 | 0.936 | 0.989 | Bb | 0.223 | 0.254 | 0.245 |
| bb | 0.850 | 0.564 | 0.866 | bb | 0.448 | 0.025 | 0.424 |
| $h^2 = 0.05, MAF = 0.2$ | | | | $h^2 = 0.05, MAF = 0.4$ | | | |
| Model epi45 | AA | Aa | aa | Model epi50 | AA | Aa | aa |
| BB | 0.638 | 0.488 | 0.383 | BB | 0.098 | 0.219 | 0.302 |
| Bb | 0.464 | 0.765 | 0.957 | Bb | 0.302 | 0.126 | 0.121 |
| bb | 0.580 | 0.562 | 0.719 | bb | 0.053 | 0.308 | 0.136 |

Table 7: Models with $h^2 = 0.05, MAF = 0.2, 0.4$.

| $h^2 = 0.025, MAF = 0.2$ | | | | $h^2 = 0.025, MAF = 0.4$ | | | |
|--------------------------|-------|-------|-------|--------------------------|-------|-------|-------|
| Model epi51 | AA | Aa | aa | Model epi56 | AA | Aa | aa |
| BB | 0.495 | 0.415 | 0.657 | BB | 0.002 | 0.155 | 0.214 |
| Bb | 0.429 | 0.616 | 0.121 | Bb | 0.199 | 0.071 | 0.022 |
| bb | 0.552 | 0.331 | 0.419 | bb | 0.081 | 0.122 | 0.135 |
| $h^2 = 0.025, MAF = 0.2$ | | | | $h^2 = 0.025, MAF = 0.4$ | | | |
| Model epi52 | AA | Aa | aa | Model epi57 | AA | Aa | aa |
| BB | 0.592 | 0.691 | 0.743 | BB | 0.188 | 0.020 | 0.171 |
| Bb | 0.712 | 0.493 | 0.419 | Bb | 0.032 | 0.174 | 0.059 |
| bb | 0.580 | 0.746 | 0.504 | bb | 0.134 | 0.087 | 0.092 |
| $h^2 = 0.025, MAF = 0.2$ | | | | $h^2 = 0.025, MAF = 0.4$ | | | |
| Model epi53 | AA | Aa | aa | Model epi58 | AA | Aa | aa |
| BB | 0.108 | 0.194 | 0.186 | BB | 0.005 | 0.179 | 0.251 |
| Bb | 0.196 | 0.037 | 0.045 | Bb | 0.211 | 0.100 | 0.026 |
| bb | 0.172 | 0.073 | 0.130 | bb | 0.156 | 0.098 | 0.156 |
| $h^2 = 0.025, MAF = 0.2$ | | | | $h^2 = 0.025, MAF = 0.4$ | | | |
| Model epi54 | AA | Aa | aa | Model epi59 | AA | Aa | aa |
| BB | 0.112 | 0.186 | 0.128 | BB | 0.174 | 0.321 | 0.154 |
| Bb | 0.193 | 0.024 | 0.138 | Bb | 0.223 | 0.254 | 0.245 |
| bb | 0.079 | 0.236 | 0.251 | bb | 0.448 | 0.025 | 0.424 |
| $h^2 = 0.025, MAF = 0.2$ | | | | $h^2 = 0.025, MAF = 0.4$ | | | |
| Model epi55 | AA | Aa | aa | Model epi60 | AA | Aa | aa |
| BB | 0.272 | 0.192 | 0.185 | BB | 0.098 | 0.219 | 0.302 |
| Bb | 0.172 | 0.367 | 0.390 | Bb | 0.302 | 0.126 | 0.121 |
| bb | 0.345 | 0.069 | 0.005 | bb | 0.053 | 0.308 | 0.136 |

Table 8: Models with $h^2 = 0.025, MAF = 0.2, 0.4$.

3 Results on WTCCC data

| Disease | SNP Groups | Location | Related Genes | Individual P-Values | Interaction P-value |
|-----------------|----------------------------------|----------|-----------------------------|---------------------|-------------------------|
| Bipolar disease | (rs4844637,rs4844639) | Chr1 | (PLXNA2,PLXNA2) | (0.001,0.774) | 1.110×10^{-16} |
| | (rs10937420,rs1522930) | Chr3 | (LEPREL1,LEPREL1) | (0.005,0.89) | 4.441×10^{-15} |
| | (rs6438214,rs10511335) | Chr3 | (LOC100132655,LOC100132655) | (0.004,0.987) | 1.193×10^{-23} |
| | (rs1396113,rs9993195) | Chr4 | (APBB2,APBB2) | (0.124,0.007) | 2.509×10^{-24} |
| | (rs11733953,rs16893604) | Chr4 | (LDB2,LDB2) | (0.007,0.311) | 6.661×10^{-16} |
| | (rs1422672,rs10515786) | Chr5 | (EBF1,EBF1) | (0.003,0.568) | 7.447×10^{-27} |
| | (rs2992406,rs2935260) | Chr5 | (CDC20B,CDC20B) | (0.009,0.689) | 5.742×10^{-26} |
| | (rs1552835,rs17519558) | Chr5 | (GRIA1,GRIA1) | (0.027,0.441) | 1.665×10^{-15} |
| | (rs2768558,rs2768541) | Chr6 | (ARMC2,ARMC2) | (0.767,0.004) | 1.039×10^{-25} |
| | (rs9320174,rs13218960) | Chr6 | (AIM1,AIM1) | (0.001,0.131) | 4.441×10^{-16} |
| | (rs10266006,rs3793181) | Chr7 | (NCAPG2,NCAPG2) | (0.039,0.129) | 2.350×10^{-23} |
| | (rs1408526,rs7851968) | Chr9 | (DFNB31,DFNB31) | (0.042,0.647) | 1.110×10^{-16} |
| | (rs4242632,rs7857957) | Chr9 | (NTRK2,NTRK2) | (0.112,0.498) | 6.217×10^{-15} |
| | (rs11010228,rs4934551) | Chr10 | (CCNY,CCNY) | (0.02,0.123) | 1.332×10^{-15} |
| | (rs1530116,rs4372362) | Chr10 | (BRWD2,BRWD2) | (0.49,0.004) | 1.110×10^{-16} |
| | (rs6479769,rs7075575) | Chr10 | (A1CF,A1CF) | (0.863,0.001) | 6.661×10^{-16} |
| | (rs12244405,rs7075575) | Chr10 | (A1CF,A1CF) | (0.878,0.001) | 2.220×10^{-15} |
| | (rs12260411,rs11250624) | Chr10 | (ADARB2,ADARB2) | (0.02,0.247) | 1.221×10^{-15} |
| | (rs2422179,rs12790641) | Chr11 | (LDLRAD3,LDLRAD3) | (0.004,0.15) | 3.331×10^{-16} |
| | (rs12424737,rs11111782) | Chr12 | (NT5DC3,NT5DC3) | (0.018,0.02) | 1.110×10^{-16} |
| | (rs7963772,rs373529) | Chr12 | (SCN8A,SCN8A) | (0.409,0.003) | 7.105×10^{-15} |
| | (rs4882804,rs4597130) | Chr12 | (TMEM132C,TMEM132C) | (0.816,0.001) | 2.220×10^{-16} |
| | (rs730747,rs9533431) | Chr13 | (ENOX1,ENOX1) | (0.249,0.061) | 4.219×10^{-14} |
| | (rs10483596,rs7152370,rs4901281) | Chr14 | (UNKNOWN,UNKNOWN,TXNDC16) | (0.156,0.006,0.011) | 2.940×10^{-34} |
| | (rs16969478,rs16969475) | Chr15 | (FSIP1,FSIP1) | (0.002,0.976) | 2.098×10^{-14} |
| | (rs8037901,rs967514) | Chr15 | (TCF12,TCF12) | (0.134,0.002) | 3.963×10^{-14} |
| | (rs2561037,rs2561036) | Chr16 | (SNX29,SNX29) | (0.01,0.003) | 1.454×10^{-38} |
| | (rs8056538,rs7186084) | Chr16 | (CDH1,CDH1) | (0.381,0.011) | 3.919×10^{-14} |
| | (rs10408464,rs3746210) | Chr19 | (ZNF211,ZNF211) | (0.663,0.005) | 1.406×10^{-13} |
| | (rs3746210,rs9749085) | Chr19 | (ZNF211,LOC100130710) | (0.005,0.842) | 9.420×10^{-13} |
| | (rs2079033,rs3746210) | Chr19 | (ZNF134,ZNF211) | (0.005,0.005) | 5.750×10^{-23} |
| | (rs17310782,rs6060163) | Chr20 | (MYH7B,TRPC4AP) | (0.002,0.249) | 1.800×10^{-23} |
| | (rs2295701,rs6060163) | Chr20 | (TRPC4AP,TRPC4AP) | (0.009,0.249) | 2.220×10^{-16} |
| | (rs6088660,rs6060163) | Chr20 | (GSS,TRPC4AP) | (0.006,0.249) | 3.109×10^{-15} |
| | (rs1206808,rs1206809) | Chr20 | (EYA2,EYA2) | (0.048,0.343) | 1.443×10^{-14} |
| | (rs1885119,rs6060163) | Chr20 | (TRPC4AP,TRPC4AP) | (0.04,0.249) | 8.515×10^{-14} |
| | (rs220503,rs6027163) | Chr20 | (C20ORF95,C20ORF95) | (0.036,0.344) | 1.427×10^{-23} |
| | (rs10854233,rs1487328) | Chr20 | (WFDC11,WFDC9) | (0.036,0.572) | 9.992×10^{-14} |
| Crohn's disease | (rs2104962,rs1931363,rs5744303) | Chr1 | (UNKNOWN,CLCA2,CLCA1) | (0.022,0.528,0.499) | 4.154×10^{-34} |
| | (rs2104962,rs272504,rs5744303) | Chr1 | (UNKNOWN,ODF2L,CLCA1) | (0.022,0.441,0.499) | 6.899×10^{-33} |
| | (rs2104962,rs6692784,rs5744303) | Chr1 | (UNKNOWN,UNKNOWN,CLCA1) | (0.022,0.023,0.499) | 5.895×10^{-35} |
| | (rs1531939,rs9826424) | Chr3 | (GRM7,GRM7) | (0.002,0.301) | 2.665×10^{-15} |
| | (rs584852,rs7897278) | Chr10 | (SORCS1,SORCS1) | (0.003,0.178) | 1.872×10^{-23} |
| | (rs7154773,rs10130695) | Chr14 | (PPM1A,PPM1A) | (0.004,0.476) | 4.435×10^{-43} |
| | (rs17569609,rs4998386) | Chr16 | (GRIN2A,GRIN2A) | (0.335,0.018) | 3.542×10^{-14} |
| | (rs783239,rs6501727) | Chr17 | (C17ORF77,LOC100133702) | (0.007,0.055) | 4.862×10^{-23} |
| | (rs11867262,rs8065239) | Chr17 | (AATF,AATF) | (0.002,0.918) | 8.089×10^{-26} |
| | (rs9896922,rs228879) | Chr17 | (PRKCA,PRKCA) | (0.096,0.027) | 1.890×10^{-13} |
| | (rs1006746,rs6016951) | Chr20 | (PTPRT,PTPRT) | (0.652,0.004) | 4.319×10^{-14} |
| | (rs2837821,rs9647188) | Chr21 | (LOC100134046,LOC100134046) | (0.896,0.003) | 3.133×10^{-13} |
| | (rs8138080,rs739296) | Chr22 | (WBP2NL,SEPT3) | (0.172,0.134) | 9.326×10^{-15} |

| Disease | SNP Groups | Location | Related Genes | Individual P-Values | Interaction P-value |
|----------------------|-------------------------|----------|-----------------------------|---------------------|-------------------------|
| Rheumatoid arthritis | (rs2809345,rs2808250) | Chr1 | (DDX59,DDX59) | (0.012,0.299) | 4.772×10^{-26} |
| | (rs2273936,rs748405) | Chr1 | (C1ORF142,C1ORF142) | (0.003,0.593) | 2.331×10^{-15} |
| | (rs4844637,rs4844639) | Chr1 | (PLXNA2,PLXNA2) | (0.002,0.678) | 4.219×10^{-15} |
| | (rs6545864,rs7579073) | Chr2 | (LOC729723,LOC729723) | (0.011,0.007) | 5.605×10^{-29} |
| | (rs6545864,rs893589) | Chr2 | (LOC729723,EFR3B) | (0.011,0.104) | 1.396×10^{-26} |
| | (rs6545864,rs955855) | Chr2 | (LOC729723,RBJ) | (0.011,0.015) | 4.135×10^{-26} |
| | (rs1605705,rs2136152) | Chr3 | (GRM7,GRM7) | (0.014,0.038) | 2.630×10^{-30} |
| | (rs9870678,rs10866009) | Chr3 | (DNAH12L,DNAH12L) | (0.072,0.138) | 2.663×10^{-24} |
| | (rs2610201,rs2610204) | Chr4 | (GALNT17,GALNT17) | (0.002,0.747) | 5.773×10^{-15} |
| | (rs1422672,rs10515786) | Chr5 | (EBF1,EBF1) | (0.012,0.386) | 1.110×10^{-16} |
| | (rs2563335,rs2245643) | Chr5 | (WDR55,IK) | (0.76,0.004) | 1.221×10^{-15} |
| | (rs801168,rs2245643) | Chr5 | (ZMAT2,IK) | (0.785,0.004) | 3.442×10^{-15} |
| | (rs801399,rs2245643) | Chr5 | (IK,IK) | (0.803,0.004) | 7.438×10^{-15} |
| | (rs801167,rs2245643) | Chr5 | (ZMAT2,IK) | (0.932,0.004) | 1.554×10^{-15} |
| | (rs2803101,rs2846530) | Chr6 | (PARK2,PARK2) | (0.1,0.002) | 2.220×10^{-16} |
| | (rs9394893,rs13205512) | Chr6 | (TRERF1,TRERF1) | (0.454,0.589) | 4.275×10^{-23} |
| | (rs9293855,rs1891698) | Chr6 | (RIMS1,RIMS1) | (0.738,0.074) | 6.560×10^{-24} |
| | (rs10216301,rs860010) | Chr7 | (CALN1,CALN1) | (0.26,0.008) | 2.887×10^{-15} |
| | (rs4732651,rs2237812) | Chr8 | (EXTL3,EXTL3) | (0.12,0.319) | 1.728×10^{-25} |
| | (rs7085631,rs10883586) | Chr10 | (TD1,TD1) | (0.005,0.002) | 1.669×10^{-33} |
| | (rs2961593,rs2924264) | Chr10 | (AKR1CL2,AKR1CL2) | (0.228,0.005) | 2.531×10^{-26} |
| | (rs12424737,rs11111782) | Chr12 | (NT5DC3,NT5DC3) | (0.022,0.038) | 1.887×10^{-15} |
| | (rs6490403,rs7997274) | Chr13 | (KIAA0774,KIAA0774) | (0.002,0.387) | 9.481×10^{-25} |
| | (rs9603602,rs9315724) | Chr13 | (COG6,COG6) | (0.071,0.428) | 2.998×10^{-15} |
| | (rs7154773,rs10130695) | Chr14 | (PPM1A,PPM1A) | (0.112,0.274) | 1.485×10^{-42} |
| | (rs1420247,rs4238755) | Chr16 | (LOC100132440,LOC100132440) | (0.001,0.929) | 1.110×10^{-16} |
| | (rs8058964,rs12598771) | Chr16 | (CDH13,CDH13) | (0.002,0.889) | 3.061×10^{-26} |
| | (rs2561037,rs2561036) | Chr16 | (SNX29,SNX29) | (0.037,0.955) | 3.331×10^{-16} |
| | (rs901064,rs7503807) | Chr17 | (KIAA1303,KIAA1303) | (0.034,0.016) | 8.831×10^{-38} |
| | (rs2343244,rs9908879) | Chr17 | (RAB11FIP4,RAB11FIP4) | (0.952,0.044) | 1.552×10^{-13} |
| | (rs11867262,rs8065239) | Chr17 | (AATF,AATF) | (0.071,0.836) | 8.588×10^{-30} |
| | (rs3850528,rs6141601) | Chr20 | (EPB41L1,EPB41L1) | (0.001,0.235) | 4.642×10^{-25} |
| | (rs1006746,rs6016951) | Chr20 | (PTPRT,PTPRT) | (0.178,0.003) | 3.997×10^{-15} |
| | (rs4911287,rs2070320) | Chr20 | (BPIL3,BPIL3) | (0.522,0.333) | 2.620×10^{-14} |
| | (rs2836318,rs2836325) | Chr21 | (LOC100131955,LOC100131955) | (0.004,0.027) | 5.285×10^{-14} |
| | (rs11090066,rs739296) | Chr22 | (WBP2NL,SEPT3) | (0.499,0.006) | 2.839×10^{-26} |

Table 9: Some significant SNP groups identified by SNPRuler on WTCCC data.

4 The deduction of utility function

| | $\zeta = 0$ | $\zeta \neq 0$ | Total |
|-----------------------|-------------|----------------|-----------------|
| $\mathcal{N}(r)$ | a | b | $a + b$ |
| $\mathcal{N}(\neg r)$ | c | d | $c + d$ |
| Total | $a + c$ | $b + d$ | $a + b + c + d$ |

Table 10: The contingency table of a given rule (r, ζ) from observations. There are a controls and b cases which satisfy r , and c controls and d cases which satisfy $\neg r$ (complementary of r). The total number of observations is $a + b + c + d$.

$$\chi^2(r, \zeta) = \frac{(a + b + c + d)(ad - bc)^2}{(a + b)(c + d)(a + c)(b + d)} \quad (1)$$

$$= \frac{(a(b + d) - b(a + c))^2}{(a + b)(c + d)} \cdot \frac{(a + b + c + d)}{(a + c)(b + d)} \quad (2)$$

$$= \frac{(\frac{b+d}{a+c} - b/a)^2}{(1 + b/a)((a + b + c + d)/a - b/a - 1)} \cdot \frac{(a + b + c + d)(a + c)}{(b + d)} \quad (3)$$

$$= \frac{(R - \delta)^2}{(1 + \delta)(\gamma - \delta - 1)} \cdot \frac{(a + b + c + d)(a + c)}{(b + d)} \quad (4)$$

$$= U(r, \zeta) \cdot \frac{(a + b + c + d)(a + c)}{(b + d)} \quad (5)$$

$$(6)$$

where $R = \frac{(b+d)}{(a+c)}$, $\delta = b/a$ and $\gamma = \frac{a+b+c+d}{a}$

5 Software instruction

- Input file format. The genotype data can either be stored in a single file or two files for case data and control data, respectively. The input file format is illustrated as below,

$$D = \begin{array}{c} T_1 \\ T_2 \\ \dots \\ T_{m-1} \\ T_m \end{array} \begin{bmatrix} S_1 & S_2 & \dots & S_n & CLASS \\ 1 & 2 & \dots & 2 & 1 \\ 0 & 1 & \dots & 1 & 0 \\ \dots & \dots & & \dots & \dots \\ 2 & 0 & \dots & 1 & 1 \\ 1 & 1 & \dots & 0 & 0 \end{bmatrix},$$

where the genotype of SNP is represented with $\{0, 1, 2\}$ corresponding to homozygous reference genotype (AA), heterozygous genotype (Aa), and homozygous variant genotype (aa), respectively. The class label is either 0 (control) or 1 (case).

- Input parameters. To run the program, input the command “java -jar rule.jar listSize depth updateRatio dataFile” or “java -jar rule.jar listSize depth updateRatio controlDataFile caseDataFile”. Here **listSize** is the expected number of interactions, **depth** is the order of interaction, and **updateRatio** is the step size of updating a rule (which usually takes a value between 0 and 1. The default value is 0.2).

References

- [1] Marchini J., Donnelly P., Cardon L.R. (2005) Genome-wide strategies for detecting multiple loci that influence complex diseases, *Nature Genetics*, **37**,413-417.
- [2] Velez D.R., White B.C., Motsinger A.A., Bush W.S., Ritchie M.D., Williams S.M. and Moore J.H. Moore (2007) A balanced accuracy function for epistasis modeling in imbalanced datasets using multifactor dimensionality reduction, *Genet Epidemiol*, **31**, 306-315.
- [3] Wang W.Y., Barratt B.J., Clayton D.G. and Todd J.A. (2005) Genome-wide association studies: theoretical and practical concerns, *Nat. Rev. Genet.*, **6**, 109-118.
- [4] Zhang Y., Liu J.S. (2007) Bayesian inference of epistatic interactions in case-control studies, *Nature Genetics*, **39**(9),1167-1173.